```
> restart;
```
Calcul des équations du mouvement

Equation du mouvement

```
> edm:=(m1+m2)*l1*diff(theta1(t),t$2)+m2*l2*diff(theta2(t),t$2)*cos(theta1(t)-theta2(t))+m2*l2*diff(theta2(t),t)^2*sin(theta1(t)-theta2(t))+(m1+m2)*g*sin(theta1(t))+c*diff(theta1(t),t)=0;
```

$$edm := (m1 + m2)\, l1 \left(\frac{d^2}{dt^2}\, \theta 1(t)\right) + m2\, l2 \left(\frac{d^2}{dt^2}\, \theta 2(t)\right) \cos(\theta 1(t) - \theta 2(t)) + m2\, l2 \left(\frac{d}{dt}\, \theta 2(t)\right)^2 \sin(\theta 1(t) - \theta 2(t)) + (m1 + m2)\, g \sin(\theta 1(t)) + c \left(\frac{d}{dt}\, \theta 1(t)\right) = 0 \tag{1.1}$$

On force la masse m2 à tourner

```
> forcing:=theta2(t)=omega*t;
```

$$forcing := \theta 2(t) = \omega\, t \tag{1.2}$$

```
> eq1:=combine(simplify(expand(subs(forcing,edm))),trig);
```

$$eq1 := l1 \left(\frac{d^2}{dt^2}\, \theta 1(t)\right) m1 + l1 \left(\frac{d^2}{dt^2}\, \theta 1(t)\right) m2 - m2\, l2\, \omega^2 \sin(-\theta 1(t) + \omega\, t) + g \sin(\theta 1(t))\, m1 + g \sin(\theta 1(t))\, m2 + c \left(\frac{d}{dt}\, \theta 1(t)\right) = 0 \tag{1.3}$$

Simplification pour des petits déplacements de theta1

```
> eq2:=expand(subs(theta1(t)=epsilon*theta1(t),eq1));
```

$$eq2 := l1\, \varepsilon \left(\frac{d^2}{dt^2}\, \theta 1(t)\right) m1 + l1\, \varepsilon \left(\frac{d^2}{dt^2}\, \theta 1(t)\right) m2 + m2\, l2\, \omega^2 \sin(\varepsilon\, \theta 1(t)) \cos(\omega\, t) - m2\, l2\, \omega^2 \cos(\varepsilon\, \theta 1(t)) \sin(\omega\, t) + g \sin(\varepsilon\, \theta 1(t))\, m1 + g \sin(\varepsilon\, \theta 1(t))\, m2 + c\, \varepsilon \left(\frac{d}{dt}\, \theta 1(t)\right) = 0 \tag{1.4}$$

```
> convert(series(lhs(eq2),epsilon,2)=0,polynom):
  eq3:=subs(epsilon=1,%);
```

$$eq3 := -m2\, l2\, \omega^2 \sin(\omega\, t) + l1 \left(\frac{d^2}{dt^2}\, \theta 1(t)\right) m1 + l1 \left(\frac{d^2}{dt^2}\, \theta 1(t)\right) m2 + m2\, l2\, \omega^2\, \theta 1(t) \cos(\omega\, t) + g\, \theta 1(t)\, m1 + g\, \theta 1(t)\, m2 + c \left(\frac{d}{dt}\, \theta 1(t)\right) = 0 \tag{1.5}$$

Calcul de la réponse du pendule (sans le terme paramétrique)

```
> eq4:=eq3-(m2*l2*omega^2*theta1(t)*cos(omega*t)=0);
```

$$eq4 := -m2\, l2\, \omega^2 \sin(\omega\, t) + l1 \left(\frac{d^2}{dt^2}\, \theta 1(t)\right) m1 + l1 \left(\frac{d^2}{dt^2}\, \theta 1(t)\right) m2 + g\, \theta 1(t)\, m1 + g\, \theta 1(t)\, m2 + c \left(\frac{d}{dt}\, \theta 1(t)\right) = 0 \tag{1.6}$$

Frequence de résonance

Fréquence de résonance

```
> eq5:=subs(sin(omega*t)=0,c=0,eq4);
```

$$eq5 := l1\left(\frac{d^2}{dt^2}\theta 1(t)\right) m1 + l1\left(\frac{d^2}{dt^2}\theta 1(t)\right) m2 + g\,\theta 1(t)\, m1 + g\,\theta 1(t)\, m2 = 0 \quad \textbf{(2.1)}$$

Equation caractéristique

```
> eqcaract:=simplify(expand(subs(theta1(t)=Theta1*exp(I*lambda*
  t),eq5)/exp(I*lambda*t)));
```

$$eqcaract := -\Theta 1\, l1\, \lambda^2\, m1 - \Theta 1\, l1\, \lambda^2\, m2 + \Theta 1\, g\, m1 + \Theta 1\, g\, m2 = 0 \quad \textbf{(2.2)}$$

```
> natfreq:=omega0=solve(eqcaract,lambda)[1];
```

$$natfreq := \omega 0 = \frac{\sqrt{l1\, g}}{l1} \quad \textbf{(2.3)}$$

Réponse du pendule forcé

```
> repform:=theta1(t)=A*cos(omega*t)+B*sin(omega*t);
```

$$repform := \theta 1(t) = A\cos(\omega t) + B\sin(\omega t) \quad \textbf{(3.1)}$$

```
> eq6:=expand(subs(repform,eq4));
```

$$eq6 := -m2\, l2\, \omega^2 \sin(\omega t) - l1\, m1\, A\cos(\omega t)\,\omega^2 - l1\, m1\, B\sin(\omega t)\,\omega^2 - l1\, m2\, A\cos(\omega t)\,\omega^2 - l1\, m2\, B\sin(\omega t)\,\omega^2 + g\, m1\, A\cos(\omega t) + g\, m1\, B\sin(\omega t) + g\, m2\, A\cos(\omega t) + g\, m2\, B\sin(\omega t) - c\, A\sin(\omega t)\,\omega + c\, B\cos(\omega t)\,\omega = 0 \quad \textbf{(3.2)}$$

Equilibrage harmonique

```
> eqCos:=coeff(lhs(eq6),cos(omega*t))=0;
```

$$eqCos := -A\, l1\, m1\, \omega^2 - A\, l1\, m2\, \omega^2 + A\, g\, m1 + A\, g\, m2 + B\, c\, \omega = 0 \quad \textbf{(3.3)}$$

```
> eqSim:=coeff(lhs(eq6),sin(omega*t))=0;
```

$$eqSim := -B\, l1\, m1\, \omega^2 - B\, l1\, m2\, \omega^2 - l2\, m2\, \omega^2 - A\, c\, \omega + B\, g\, m1 + B\, g\, m2 = 0 \quad \textbf{(3.4)}$$

On résoud pour A et B

```
> eq7:=solve([eqCos,eqSim],[A,B])[];
```

$$eq7 := \left[A = -\left(c\, l2\, m2\, \omega^3\right) \Big/ \left(l1^2\, m1^2\, \omega^4 + 2\, l1^2\, m1\, m2\, \omega^4 + l1^2\, m2^2\, \omega^4 - 2\, g\, l1\, m1^2\, \omega^2 - 4\, g\, l1\, m1\, m2\, \omega^2 - 2\, g\, l1\, m2^2\, \omega^2 + c^2\, \omega^2 + g^2\, m1^2 + 2\, g^2\, m1\, m2 + g^2\, m2^2\right), B = \left(\left(-l1\, m1\, \omega^2 - l1\, m2\, \omega^2 + g\, m1 + g\, m2\right) l2\, m2\, \omega^2\right) \Big/ \left(l1^2\, m1^2\, \omega^4 + 2\, l1^2\, m1\, m2\, \omega^4 + l1^2\, m2^2\, \omega^4 - 2\, g\, l1\, m1^2\, \omega^2 - 4\, g\, l1\, m1\, m2\, \omega^2 - 2\, g\, l1\, m2^2\, \omega^2 + c^2\, \omega^2 + g^2\, m1^2 + 2\, g^2\, m1\, m2 + g^2\, m2^2\right)\right] \quad \textbf{(3.5)}$$

Amplitude de la réponse

```
> theta1max:=simplify(expand(subs(eq7,sqrt(A^2+B^2))));
```

$$theta1max := \left(\left(l2^2\, m2^2\, \omega^4\right) \Big/ \left(l1^2\, m1^2\, \omega^4 + 2\, l1^2\, m1\, m2\, \omega^4 + l1^2\, m2^2\, \omega^4 - 2\, g\, l1\, m1^2\, \omega^2 - 4\, g\, l1\, m1\, m2\, \omega^2 - 2\, g\, l1\, m2^2\, \omega^2 + c^2\, \omega^2 + g^2\, m1^2 + 2\, g^2\, m1\, m2 + g^2\, m2^2\right)\right)^{1/2} \quad \textbf{(3.6)}$$

Application numérique

```
> param1:=[l1=4,m1=6,m2=0.5,l2=0.2,g=9.81];
```

$$param1 := [l1=4, m1=6, m2=0.5, l2=0.2, g=9.81] \quad (4.1)$$

```
> evalf(subs(param1,natfreq));
  f0=evalf(rhs(%)/2/Pi);
```

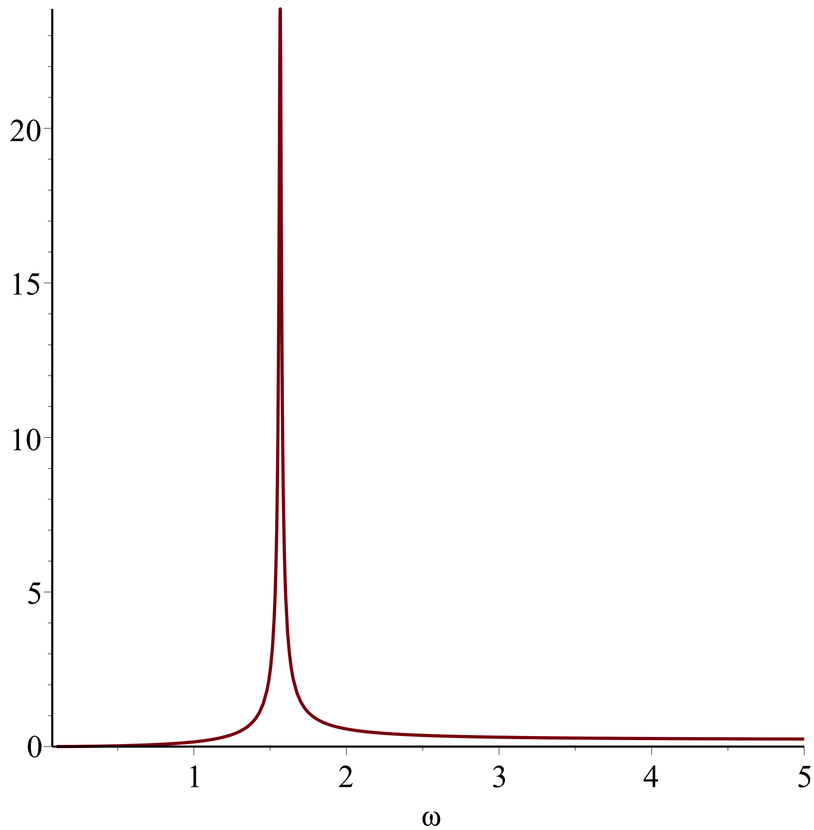
$$\omega 0 = 1.566045976$$

$$f0 = 0.2492439581 \quad (4.2)$$

```
> param:=[param1[],subs(param1,c=2*m1*0.02*rhs(natfreq))];
```

$$param := [l1=4, m1=6, m2=0.5, l2=0.2, g=9.81, c=0.3758510343] \quad (4.3)$$

```
> subs(param,theta1max):
  plot(%*180/Pi,omega=0.1..5);
```



Déplacement horizontal

```
> Projection:=[Hx=l1*sin(theta1max),Hy=l1-l1*cos(theta1max)];
```

$$Projection := [Hx \quad (4.4)$$

$= l1$

$$\sin\left(\left(\left(l2^2\, m2^2\, \omega^4\right) / \left(l1^2\, m1^2\, \omega^4 + 2\, l1^2\, m1\, m2\, \omega^4 + l1^2\, m2^2\, \omega^4 - 2\, g\, l1\, m1^2\, \omega^2 - 4\, g\, l1\, m1\, m2\, \omega^2 - 2\, g\, l1\, m2^2\, \omega^2 + c^2\, \omega^2 + g^2\, m1^2 + 2\, g^2\, m1\, m2 + g^2\, m2^2\right)\right)^{1/2}\right), Hy$$

$= l1$

$- l1$

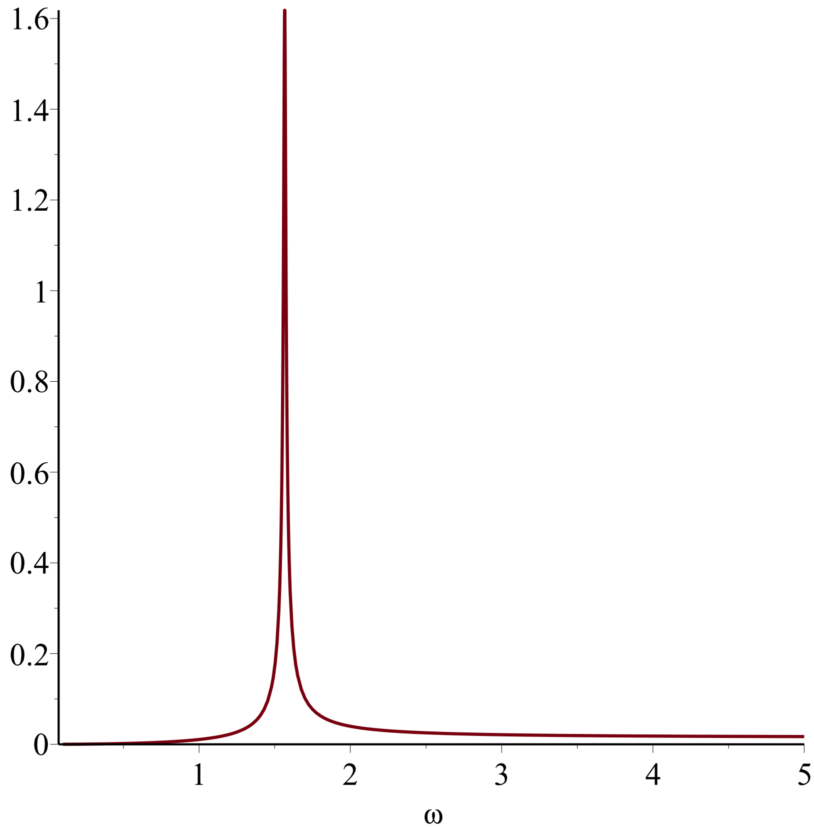
$$\cos\left(\left(\left(l2^2\, m2^2\, \omega^4\right) / \left(l1^2\, m1^2\, \omega^4 + 2\, l1^2\, m1\, m2\, \omega^4 + l1^2\, m2^2\, \omega^4 - 2\, g\, l1\, m1^2\, \omega^2 - 4\, g\, l1\, m1\, m2\, \omega^2 - 2\, g\, l1\, m2^2\, \omega^2 + c^2\, \omega^2 + g^2\, m1^2 + 2\, g^2\, m1\, m2 + g^2\, m2^2\right)\right)^{1/2}\right)\Big]$$

```
> temp:=subs(param,Projection);
```

$$temp := \left[Hx = 4 \sin\left(0.100 \sqrt{\frac{\omega^4}{676.00\, \omega^4 - 3315.638736\, \omega^2 + 4065.975225}}\right), Hy = 4 - 4 \cos\left(0.100 \sqrt{\frac{\omega^4}{676.00\, \omega^4 - 3315.638736\, \omega^2 + 4065.975225}}\right)\right] \quad \textbf{(4.5)}$$

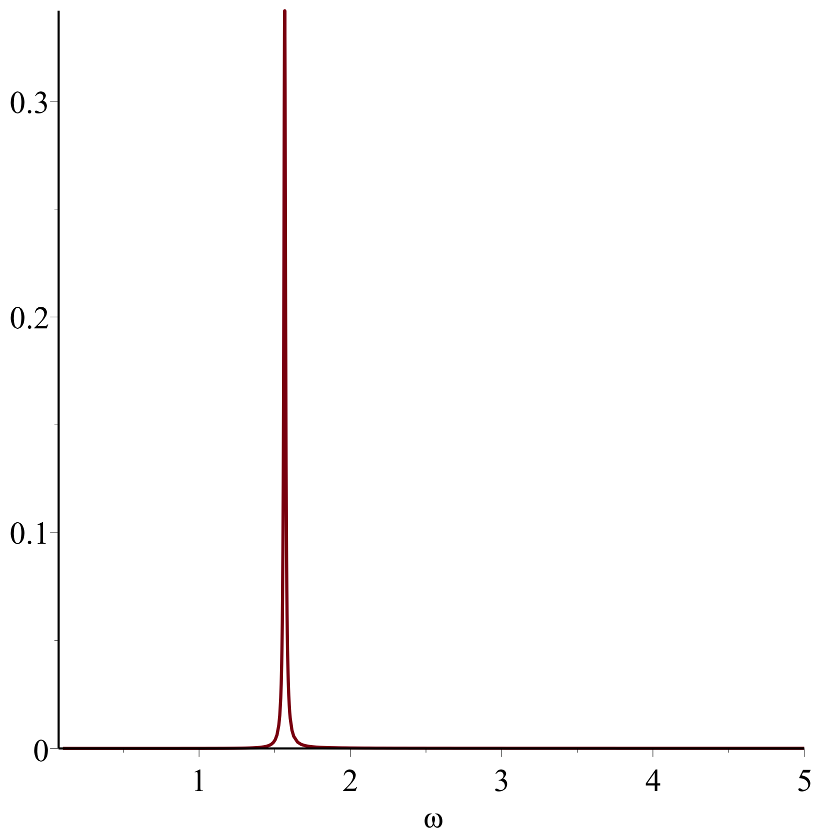
Deplacement horizontal

```
> plot(rhs(temp[1]),omega=0.1..5);
```



Deplacement vertical

```
> plot(rhs(temp[2]),omega=0.1..5);
```


0.3
0.2
0.1
0
1
2
3
4
5
ω